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International poverty projections

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Abstract

This paper is an investigation of the methodology of international poverty projections, particularly those that have formed the basis of many World Bank documents. The methodology, as developed by Ahluwalia, Carter, and Chenery (1979) in a widely-cited paper, is examined critically and subjected to sensitivity analysis. We find that their projections of poverty are not robust to reasonable changes and improvements in the methodology: in some cases even the time trend of the projections is reversed. Analysts and policy-makers should, therefore, treat such global poverty forecasts with due caution.

1. Introduction

In his foreword to the first *World Development Report* of the World Bank (1978), Robert McNamara wrote (p. iii):

The past quarter century has been a period of unprecedented change and progress in the developing world. And yet despite this impressive record, some 800 million individuals continue to be trapped in what I have termed absolute poverty…

Absolute poverty on so massive a scale is already a cruel anachronism. But unless economic growth in the developing countries can be substantially accelerated, the now inevitable increases in population will mean that the numbers of absolute poor will remain unacceptably high even at the end of the century.

This paper was presented at a conference on 'Poverty, Undernutrition and Living Standards' held at WIDER, Helsinki in July 1987. We are grateful to WIDER for financial support in undertaking this research (Anand and Kanbur 1985), and to Carine Ronsmans and Amartya Sen for comments. The research was begun during the final stages of our ESRC project 'Inequality and Development: A Reconsideration' (grant no. B 0023 0001).

The twin objectives of development, then, are to accelerate economic growth and to reduce poverty.

A full chapter of the *World Development Report 1978* was devoted to examining the prospects for growth and the alleviation of poverty. Alternative projections of growth in the developing countries have been constructed from various assumptions and scenarios about their internal policies and external circumstances. The impact of such growth on absolute poverty has then been traced by means of a simulation model. This model ...

> combines the GNP growth rates projected for different groups of countries with the assumption that the inequality of incomes is likely to increase in the early stages of development, and then to decrease in the later stages of development... This assumption can be supported by tests based on cross-country comparisons relating measures of income equality to the average income levels in each country... Assuming that the rates of growth projected for the period 1975-85 hold to the end of this century, and assuming the relation between income distribution and aggregate growth just described, the proportion of population living in absolute poverty in the year 2000 is projected as shown in (the) table...
>
> (World Bank 1978, p. 33).[1]

The assumption mentioned above that inequality first increases and then decreases with development is, of course, the now-famous 'inverse-U' hypothesis due to Kuznets (1955). (A formalization of Kuznets' analysis is contained in Anand and Kanbur 1984*b*.) Support for the assumption through 'tests based on cross-country comparisons' refers mainly to the influential paper of Ahluwalia (1976).[2] This paper has become the centerpiece of the recent literature on inequality and development, and—apart from being widely cited[3]—it has been reprinted in collections of readings in development economics (e.g. Livingstone

[1] The *World Development Report 1979* also contains estimates of absolute poverty in the year 2000 under alternative scenarios (World Bank 1979, p. 19). The entire Part II of the *World Development Report 1980* is devoted to the theme of poverty and human development. It, too, contains estimates of absolute poverty in developing countries, 'taking as the cutoff a level of income based on detailed studies of poverty in India... ' (World Bank 1980, p. 33). See Ahluwalia, Carter and Chenery (1979, pp. 304–305).

[2] See also Adelman and Morris (1973) and Paukert (1973).

[3] For example, Srinivasan (1977, pp. 14–15) lends qualified support to Ahluwalia's cross-sectional estimates, adding that 'it is... possible to make some limited and stylized policy simulations based on the curve.' We take this latter statement as support for the simulations in Ahluwalia, Carter and Chenery (1979).

1981). Not only has the Ahluwalia paper served to 'confirm' the inverse-U hypothesis, but its *particular* estimation of the inequality-development relationship has been used for projections of poverty in the *World Development Reports*.[4] The technical background and methodology for these projections of poverty are contained in another authoritative paper—that by Ahluwalia, Carter and Chenery (1979), henceforth referred to as ACC.

The object of this paper is to reconsider the World Bank-ACC projections of international poverty. Specifically, our present paper attempts to evaluate the robustness of the projections to changes in the underlying assumptions of their methodology. Section 2 of the paper documents (as far as is possible) the ACC projections method. As noted above, at the heart of the method is the use of estimated Kuznets curves to project quintile shares. Section 3 notes that the ACC method of interpolating poverty from forecasts of quintile shares assumes a particular distribution *within* each quintile. Without this assumption we are only able to derive bounds for poverty. The section shows that these bounds can be wide—wide enough to reverse the trend of country poverty forecasts by the ACC method. Section 4 considers forecasts for an alternative poverty index, the poverty gap ratio. Sections 5 and 6 focus on the functional form of the estimated Kuznets curves. Section 5 re-estimates the curves taking into account the fact that quintile shares are limited-dependent variables. In contrast to Ahluwalia's (1976) use of the log-quadratic functional form, Section 6 introduces per capita income in quadratic and inverse-quadratic forms. Section 7 summarizes the main results and conclusions of the paper. Three Appendices take up some technical details. Appendix A evaluates the econometric basis of the ACC method in terms of the bias and efficiency of the forecasts. Appendix B investigates the per capita income projections underlying the ACC method. Appendix C considers the effect of using purchasing power parity conversions of per capita GNP, sometimes also called Kravis factors.

2 The ACC method

The ACC procedure of poverty projection consists of four steps:

(a) estimation of the income level of each country... for the past (1960–1975) and projection of this level for the future (1975-2000)

(b) estimation of population... by country for the same periods

[4] We have commented elsewhere on Ahluwalia's estimation of the inequality-development relationship—see Anand and Kanbur (1984*a*,*c*).

(c) estimation of income shares by deciles... for each country and hence the level of income for each decile group

(d) determination of the number of people... below the absolute poverty line in each year.

(Ahluwalia, Carter and Chenery 1979, p. 311).

The centerpiece of the method is step (c), and we will start with that. This step is itself in two parts—projection of quintile shares, and conversion of these quintile shares into decile shares. Let us take up the projection of quintile shares.

The projection of each quintile share relies on an estimated relationship between quintile share and per capita income for each of the five quintiles, and a 'base year' observation on each quintile for a country. Taking the share of the first quintile, I_{20}, as an example, if a country's observed I_{20} in the base year is above (below) the value of I_{20} predicted by the estimated relationship between I_{20} and per capita income by an amount Δ, then it is assumed that the country will remain above (below) the estimated relationship by the same amount Δ throughout. Given any projection of per capita income, therefore, I_{20} is determined for this country. The same procedure applies to other quintile shares and to other countries.

Given the above procedure, we are faced with three questions:

(i) Where does the estimated relationship between quintile share and per capita income come from?

(ii) Where does the 'base year' observation of the quintile share for each country come from? smallskip

(iii) Where does the projected per capita income come from?

We attempt to answer questions (i) and (ii) in this section; question (iii) is the subject of Appendix B.

(i) The relationship between quintile share and per capita income for each of the five quintiles is taken from estimates in Ahluwalia (1976, Table 1, p. 311). Ahluwalia regresses the income share of the lowest 20, 40, and 60 per cent, and the top 20 per cent, against log per capita GNP and the square of log per capita GNP. ACC take the relationship between the first quintile and per capita GNP, and between the fifth quintile and per capita GNP, directly from Ahluwalia. For the second (third) quintile the Ahluwalia estimated relationship between the income share of the lowest 20 per cent (40 per cent) and per capita GNP is subtracted from his estimated relationship between the income share of the lowest 40 per cent (60 per cent) and per capita GNP. The relationship between the fourth quintile and per capita GNP is estimated as a residual—by adding up the relationships for the other quintiles and subtracting from 100 per cent.

The role of the Ahluwalia (1976) estimates of the relationship between quintile shares and per capita income is made explicit by ACC (p. 334):

> An assumption that the income distributions of countries are unchanged over the 41-year time period (of the projection exercise) is unrealistic, thus it was necessary to incorporate what is known as the Kuznets curve. This posits an income distribution that changes with income per capita, worsening up to a certain income per capita and then slowly improving at levels above. Fortunately, estimations of this curve on data similar to ours have recently been made.

These 'estimations' are, of course, the Ahluwalia (1976) estimates of various income shares regressed against a quadratic in log per capita GNP (see his Table 1, p. 311).

The ACC projections of inequality are centered on these equations and assume that (p. 316)

> countries...retain their relative positions above or below the average distribution and in this sense are assumed to run 'parallel' to the Kuznets curve. Although this is a highly stylized interpretation of the existing evidence, it is more plausible than assuming that there is no effect of economic development and industrialization on distribution, which is the only obvious alternative.

In other words, the assumption is that the gap between any country's actual and estimated income share (from the relationship) remains constant with development.

This procedure would seem to derive from the advice of Srinivasan (1977, pp. 14–15) relating to 'country-specific' projections (the 'second type' in the paragraph cited below):

> The cross-sectional curve essentially represents an average relationship. The deviation of an individual country observation from the estimated curve could be viewed as the effect of the policies being followed as well as other relevant specific features of that country. Two types of projections can be made from the curve: in one, starting from any level of per capita GNP, one projects the per capita income for a future year and from the curve reads off the share of the bottom 40 per cent. Making projections in this way, one is really comparing the expected

income (hypothetical average) share of the bottom 40 per cent in countries which have the initial level of per capita GNP to the expected share in countries where income has reached the projected value. This type of projection is clearly not country-specific. In the second type of projection, one starts from the given initial income level and the initial share of the bottom 40 per cent, then one adds the change in the share as estimated from the curve to the initial share to obtain the share associated with the projected terminal income. In this exercise, some allowance is made for the country's specific initial circumstances. Projections of either type, if they mean anything at all, indicate what might happen if incomes changed but the distributional and other policy did not change significantly.

A formal statement of the ACC procedure is, thus, as follows. Let $Q_i(t)$ be a (sample) observation of the income share of a particular quintile (e.g. the first, or I_{20}) for a given country i in year t. Denote the *estimated* relationship between Q and per capita GNP, Y, as[5]

$$\widehat{Q}(Y) = \widehat{\alpha} + \widehat{\beta}(\log Y) + \widehat{\gamma}(\log Y)^2.$$

The estimates $\widehat{\alpha}$, $\widehat{\beta}$ and $\widehat{\gamma}$ are taken from the regression set (A) in Ahluwalia (1976), Table 1, p. 311.[6] Let $Y_i(t)$ and $Y_i(2000)$ be the per capita GNP of country i in years t and 2000, respectively. The ACC projection of the income share of the quintile in question in year 2000, $Q_i(2000)$, can then be written as

$$Q_i(2000) = \widehat{Q}(Y_i(2000)) + [Q_i(t) - \widehat{Q}(Y_i(t))]. \qquad (2.1)$$

In other words, the gap in the year 2000 is assumed to be the *same* as the gap in the year of the (sample) observation t. The econometric rationale underlying this procedure is analysed in Appendix A, where it is shown that it will produce unbiased projections only under certain very strong conditions.

(ii) The answer to question (ii) about the 'base year' observation $Q_i(t)$ for each country i is given in ACC Table A.1 (p. 333). They choose 36 countries for investigation. Of these 36 countries, for three countries (Ethiopia, Zaire, and Ghana), 'income distribution data was

[5] Ahluwalia's equation also contains a dummy variable for socialist countries, which ACC set to zero. Note, however, that ACC's list of countries includes Yugoslavia (Ibid., Table A.1, p. 333). This country was classified as socialist by Ahluwalia (1976) with dummy set equal to one there!

[6] As ACC (p. 334, n. 38) note: 'We have used the full sample estimates, see Ahluwalia (1976, p. 311)'; they reproduce his coefficient values on their p. 334.

not available' so a 'base period' observation could not be had. It was simply assumed that these countries followed the relationships estimated in Ahluwalia (1976) exactly, without any adjustment. For seven other countries (Burma, Uganda, Sudan, Tanzania, Nigeria, Morocco, and Guatemala) the same procedure was followed because income distribution data, though available, were deemed 'unreliable'.[7] For the remaining 26 countries, the 'base year' observation is provided by income distribution estimates taken from Jain (1975).[8]

Given the quintile share observations and the estimated relationships between quintile share and per capita income, the next step in the ACC procedure requires projections of per capita income. There are several problems and inconsistencies with their per capita income calculations; the discussion on these is relegated to Appendix B.

Given projected quintile shares for any year, ACC first convert these into decile shares by making the assumption that 'the shares of the two individual deciles in each quintile remain constant over time' (p. 336). Thus they go back to the decile shares in the 'base year' observation, and split the estimated quintile share into deciles in the projection year in the same ratio. One can ask about the basis of this procedure, and why the exercise should not be conducted directly on relationships estimated between decile shares and per capita income, but in what follows we take this step in their procedure as given.

Even given the decile shares and an absolute poverty line, there is still the question of interpolation in order to calculate the fraction of people below this poverty line. There is no discussion at all in ACC of this procedure, so we have attempted to reconstruct it by correspondence with the authors and from the Giniworld program that was sent to us.[9] Given the decile shares and the overall per capita

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[7] ACC do not discuss why the data on these countries were unreliable (for Tanzania and Uganda, they were good enough to be used in Ahluwalia's (1976) regressions which form the basis of the ACC projection exercises!).

[8] But there are many such estimates listed in Jain (1975) for these countries. ACC do not indicate the basis on which a particular distribution was chosen as the 'base period' observation. ACC Table A.1 (p. 333) reveals that the distributions are not consistent with respect to type and coverage, nor is it the case that the most recent distribution available from Jain (1975) is chosen. For India, for example, a 'Household-National' (HH-NL) distribution is available for 1967/68, yet the one chosen by ACC is for 1964/65. For one country, Iran, the income distribution observation for Venezuela is used, quite arbitrarily. Notice also that the information provided in Table A.1 (p. 333) does *not* identify a survey in Jain (1975) for Mexico, Turkey, or Korea. For these countries we have simply used the latest HH-NL survey from Jain (1975). For Korea there are two latest (1971) HH-NL distributions (7 and 8) in Jain (1975); we have used the distribution numbered 7.

[9] The correspondence from the authors, including the Giniworld program, cleared up some of our queries but left unresolved several problems to do with replicating the ACC poverty estimates (see Anand and Kanbur 1981).

income, the mean income μ_j of each decile can be calculated for $j = 1, 2, \ldots, 10$. Let z be the poverty line. Using the notation $\mu_0 = 0$, and assuming that $\mu_j < z < \mu_{j+1}$, our best guess of the ACC method for determining the headcount ratio H is the formula

$$H = \begin{cases} 5z/\mu_1 & \text{if } j = 0 \\ 10\dfrac{z - \mu_j}{(\mu_{j+1} - \mu_j)} + (10j - 5) & \text{if } j \geq 1. \end{cases}$$

We note here that a sufficient condition for this formula to be correct is that incomes in the j^{th} and $(j+1)^{\text{th}}$ deciles are uniformly distributed. We will return to this point in the next section.

All seems set for forecasting the headcount ratio, but there is one further problem—for the ten countries for which income distribution data was either unavailable or deemed unreliable, there exists no 'base period' observation on decile shares and hence no way of translating forecast quintile shares into decile shares. Yet we find that ACC do indeed have headcount ratio forecasts for these countries (Table 1, pp. 302–303; Table 2, pp. 312–313). We have been unable to decipher how these calculations were made. As a result, our forecasts and discussion are restricted to the remaining 26 countries.

We now come to the question of the poverty line, and relating this to the incomes of different deciles. ACC chose a poverty line of 200 ICP dollars (p. 304)—which are dollars converted at 'equivalent purchasing power conversion ratios' estimated by Kravis *et al.* (1978), the so-called 'Kravis factors'. (This cuts off the 46th percentile in the forecast Indian income distribution for 1975.) At official exchange rates, 200 ICP dollars translate to 68.3 U.S. dollars in 1970. We now have two options—calculate poverty using official exchange rate conversions, or calculate it using Kravis factor adjustments. We deal with Kravis factors in Appendix C of this paper. For now we continue with the 'official exchange rate' story.

Our Table 1 presents alternative estimates of the headcount ratio H for 1975. $H_{\text{ACC}}(1975)$ is reproduced from ACC Table 1 (p. 302). $H(1975)$ is our own estimate, using projections of per capita income discussed in Appendix B. Comparing $H(1975)$ with $H_{\text{ACC}}(1975)$ for the 26 countries for which a comparison can be made, we see that the discrepancy is larger than one percentage point for ten out of the 26 countries. The discrepancies are on the whole larger for countries with low headcount ratios. For some of these countries the discrepancy is as large as three or four hundred per cent. In fact, nine out of these ten cases occur where $H(1975)$ is less than 5 per cent, which suggests that interpolation at the lower end could be the problem. But the

Table 1. Alternative Estimates of the Headcount Ratio for 1975 (per cent)

	Country	$H(1975)$	$H_{ACC}(1975)$
1.	Bangladesh	60.3	60
2.	Ethiopia	...	62
3.	Burma	...	56
4.	Indonesia	62.6	62
5.	Uganda	...	45
6.	Zaire	...	49
7.	Sudan	...	47
8.	Tanzania	...	46
9.	Pakistan	33.6	34
10.	India	47.3	46
11.	Kenya	48.2	48
12.	Nigeria	...	27
13.	Philippines	28.7	29
14.	Sri Lanka	10.3	10
15.	Senegal	28.6	29
16.	Egypt	13.8	14
17.	Thailand	22.7	23
18.	Ghana	...	19
19.	Morocco	...	16
20.	Côte d'Ivoire	13.9	14
21.	Korea	3.8	6
22.	Chile	4.6	9
23.	Zambia	3.7	7
24.	Colombia	13.5	14
25.	Turkey	15.5	11
26.	Tunisia	4.7	9
27.	Malaysia	8.0	8
28.	Taiwan	1.9	4
29.	Guatemala	...	9
30.	Brazil	8.3	8
31.	Peru	14.9	15
32.	Iran	8.1	8
33.	Mexico	2.2	10
34.	Yugoslavia	1.8	4
35.	Argentina	1.7	3
36.	Venezuela	2.6	5

Note: ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

explanation for these discrepancies can lie in any number of procedural differences that we have attempted to document, and some that we have not been able to document, from the ACC paper.

ACC do not provide a direct estimate of their forecast headcount ratio for the year 2000. However, using the 1975 population figures from ACC Table 1, the 1975–2000 population growth rates from ACC Table 2, and their estimate of the numbers of people in poverty in the year 2000 from ACC Table 2, we can calculate the implied headcount ratio. This is presented as $H_{\mathrm{ACC}}(2000)$ in our Table 2.

Comparing $H_{\mathrm{ACC}}(2000)$ with $H(2000)$ in Table 2, we note a similar pattern of discrepancies as that observed between $H_{\mathrm{ACC}}(1975)$ and $H(1975)$. Of the 26 comparable countries, there is a discrepancy of more than one percentage point for no fewer than 22 countries, and the discrepancies are extremely large for some countries (for example, for Senegal, $H(2000)$ is 17.9 per cent while $H_{\mathrm{ACC}}(2000)$ is 25.7 per cent; for Argentina, $H(2000)$ is 0.6 per cent while $H_{\mathrm{ACC}}(2000)$ is 3.1 per cent). Once again, the discrepancies can arise for a number of reasons we have discussed. Given that our object is to test for the sensitivity of projections to variations in the ACC procedure, it is important that we use as our point of reference a set of *replicable* projections. For this reason, from now on we will use the $H(1975)$ and $H(2000)$ forecasts as our reference points.

3 Bounds for the headcount ratio forecasts

As noted in Section 2, if the distribution of income within the deciles j and $j+1$ (where $\mu_j < z < \mu_{j+1}$) is not identically uniform, then the ACC interpolation formula for deriving the headcount ratio from decile mean incomes is no longer necessarily accurate. If we do not make the assumption of a uniform distribution, what can be said about the headcount ratio? Let

$$\mu_{j-1} < \mu_j < z < \mu_{j+1} < \mu_{j+2}. \tag{3.1}$$

The upper bound on the headcount ratio is given by putting everybody in the j^{th} decile below the poverty line, and as many people from the $(j+1)^{\text{th}}$ decile as possible just below the poverty line. This latter fraction has to be consistent with the information given, viz. that the mean income of the $(j+1)^{\text{th}}$ decile is μ_{j+1} and that the highest possible income in the $(j+1)^{\text{th}}$ decile is μ_{j+2}. Given these constraints, the largest fraction of people in the $(j+1)^{\text{th}}$ decile who can be put just below the poverty line is given by $\overline{\pi}_{j+1}$, where $\overline{\pi}_{j+1}$ is the solution

Table 2. Alternative Estimates of the Headcount Ratio for 2000 (per cent)

	Country	$H(2000)$	$H_{ACC}(2000)$
1.	Bangladesh	34.9	37.4
2.	Ethiopia	...	48.2
3.	Burma	...	55.8
4.	Indonesia	16.9	15.1
5.	Uganda	...	52.3
6.	Zaire	...	32.4
7.	Sudan	...	22.2
8.	Tanzania	...	29.8
9.	Pakistan	13.8	18.3
10.	India	18.8	17.4
11.	Kenya	30.2	34.7
12.	Nigeria	...	19.5
13.	Philippines	6.9	7.8
14.	Sri Lanka	4.3	9.3
15.	Senegal	17.9	25.7
16.	Egypt	3.8	8.6
17.	Thailand	2.4	5.3
18.	Ghana	...	30.0
19.	Morocco	...	5.8
20.	Côte d'Ivoire	3.3	8.3
21.	Korea	0.8	2.0
22.	Chile	1.5	6.5
23.	Zambia	2.6	9.1
24.	Colombia	1.8	5.2
25.	Turkey	3.5	6.0
26.	Tunisia	1.1	0.0
27.	Malaysia	1.8	5.3
28.	Taiwan	0.6	0.0
29.	Guatemala	...	9.3
30.	Brazil	1.3	3.5
31.	Peru	5.8	7.1
32.	Iran	1.5	3.3
33.	Mexico	0.8	4.8
34.	Yugoslavia	0.4	0.0
35.	Argentina	0.6	3.1
36.	Venezuela	0.7	4.0

Note: ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

to

$$z\overline{\pi}_{j+1} + (1 - \overline{\pi}_{j+1})\mu_{j+2} = \mu_{j+1} \tag{3.2}$$

$$\overline{\pi}_{j+1} = \begin{cases} \dfrac{\mu_{j+2} - \mu_{j+1}}{\mu_{j+2} - z} & ;\ j \leq 8 \\ 1 & ;\ j \geq 9. \end{cases} \tag{3.3}$$

Thus the maximum headcount ratio $H_{\max}$ is given by

$$\begin{aligned} H_{\max} &= (0.1)j + (0.1)\overline{\pi}_{j+1} \\ &= (0.1)j + \begin{cases} (0.1)\dfrac{\mu_{j+2} - \mu_{j+1}}{\mu_{j+2} - z} & ;\ j \leq 8 \\ 0.1 & ;\ j \geq 9. \end{cases} \end{aligned} \tag{3.4}$$

Similarly, the smallest headcount ratio is obtained by putting everybody in the $(j+1)^{\text{th}}$ decile above the poverty line, which is certainly consistent with the information given, and as many people as possible from the j^{th} decile just above the poverty line, subject to the constraints imposed by the information given. The largest fraction of people from the j^{th} decile who can be allocated in this way is given by $\underline{\pi}_j$, where this is the solution to

$$z\underline{\pi}_j + (1 - \underline{\pi}_j)\mu_{j-1} = \mu_j \quad ; \qquad j \geq 1 \tag{3.5}$$

i.e.

$$\underline{\pi}_j = \begin{cases} \dfrac{\mu_j - \mu_{j-1}}{z - \mu_{j-1}} & ; \qquad j \geq 1 \\ 0 & ; \qquad j = 0. \end{cases} \tag{3.6}$$

Hence the minimum headcount ratio $H_{\min}$ is given by

$$\begin{aligned} H_{\min} &= (0.1)j - (0.1)\underline{\pi}_j \\ &= \begin{cases} (0.1)j - (0.1)\dfrac{\mu_j - \mu_{j-1}}{z - \mu_{j-1}} & ;\ j \geq 1 \\ 0 & ;\ j = 0. \end{cases} \end{aligned} \tag{3.7}$$

Table 3 gives our estimates of the headcount ratio bounds for 1975 and 2000. $H_{\min}(1975)$ and $H_{\max}(1975)$ are comparable with $H(1975)$, while $H_{\min}(2000)$ and $H_{\max}(2000)$ are comparable with $H(2000)$. As can be seen from the table, the bounds are fairly wide. One indication of the range of these bounds is the difference they can make to the conclusion with regard to the *trend* of poverty from 1975 to 2000. Comparing $H(1975)$ with $H(2000)$, we see that for every one of the

Table 3. Headcount Ratio Bounds for 1975 and 2000 (per cent)

& Country		$H_{min}(1975)$	$H_{max}(1975)$	$H_{min}(2000)$	$H_{max}(2000)$
1.	Bangladesh	53.9	67.5	24.9	40.0
2.	Ethiopia	...	...	...	...
3.	Burma	...	...	...	...
4.	Indonesia	55.3	68.2	11.3	24.5
5.	Uganda	...	...	...	...
6.	Zaire	...	...	...	...
7.	Sudan	...	...	...	...
8.	Tanzania	...	...	...	...
9.	Pakistan	24.9	38.8	4.1	18.6
10.	India	41.9	56.1	12.4	25.1
11.	Kenya	42.6	57.1	23.6	38.1
12.	Nigeria	...	...	...	...
13.	Philippines	23.4	36.1	1.6	15.9
14.	Sri Lanka	2.4	15.5	0.0	8.1
15.	Senegal	23.9	35.7	12.1	26.7
16.	Egypt	4.8	18.6	0.0	8.2
17.	Thailand	18.1	28.5	0.0	0.6
18.	Ghana	...	...	...	...
19.	Morocco	...	...	...	...
20.	Côte d'Ivoire	2.1	19.3	0.0	4.7
21.	Korea	0.0	6.6	0.0	3.6
22.	Chile	0.0	8.7	0.0	4.3
23.	Zambia	0.0	2.2	0.0	1.4
24.	Colombia	3.0	18.9	0.0	4.4
25.	Turkey	10.6	24.8	0.0	7.5
26.	Tunisia	0.0	8.4	0.0	3.0
27.	Malaysia	2.6	15.4	0.0	6.5
28.	Taiwan	0.0	4.2	0.0	3.4
29.	Guatemala	...	...	...	...
30.	Brazil	1.4	17.2	0.0	4.1
31.	Peru	7.1	19.9	1.6	15.3
32.	Iran	1.8	15.9	0.0	5.0
33.	Mexico	0.0	1.5	0.0	1.1
34.	Yugoslavia	0.0	5.7	0.0	4.9
35.	Argentina	0.0	5.6	0.0	4.9
36.	Venezuela	0.0	5.9	0.0	4.5

Note: ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

26 countries the ACC interpolation method forecasts a decline in absolute poverty. Obviously the same will be true if we compare $H_{\max}(1975)$ with $H_{\min}(2000)$. However, let us consider the other extreme by comparing $H_{\min}(1975)$ with $H_{\max}(2000)$. If we entertain this possibility, then for no fewer than 18 countries an *increase* in poverty is forecast. While representing an outer limit of possibility, such a comparison should nevertheless warn us about the problems in using simple interpolation to estimate the headcount ratio from decile mean incomes.

4 Forecasts for an alternative poverty index

The headcount ratio is one of the best known and most widely used indices of poverty. However, the index has been criticized because of its sole focus on the *numbers* in poverty, and disregard for the extent to which the incomes of the poor fall below the poverty line (Sen 1976). Thus a transfer of income from the poor to the non-poor will leave the headcount ratio unchanged. A number of suggestions have been made to take account of this shortcoming. One measure which attempts to do this is the per capita poverty gap ratio (Anand 1977; Foster *et al.* 1984). With individuals labelled in non-descending order of income y_i $(i = 1, 2, \ldots, n)$, z the poverty line income, and q the number in poverty, we have

$$y_1 \le y_2 \le \cdots \le y_q \le z < y_{q+1} \le \cdots \le y_n.$$

The poverty gap ratio, P, is simply defined as

$$P = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{z - y_i}{z} \right). \tag{4.1}$$

While the formula in (4.1) requires knowledge of the entire distribution of income below z, all we have from the ACC procedure are the (forecast) decile mean incomes. We can attempt to derive *bounds* for P. Given $\mu_j < z < \mu_{j+1}$, we can maximize P by putting everybody in decile j at the income level μ_j, and putting a fraction π from decile $(j+1)$ at the income level y such that $\mu_j \le y \le z$, the constraint being that $\pi y + (1 - \pi)\mu_{j+2} = \mu_{j+1}$. $P_{\max}$ is thus given as the solution to

$$\begin{aligned} P_{\max} = \operatorname*{Max}_{y}\ (0.1) \sum_{i=1}^{j} \left(\frac{z - \mu_i}{z} \right) + 0.1\pi \left(\frac{z - y}{z} \right) \\ \text{subject to} \quad \pi y + (1 - \pi)\mu_{j+2} = \mu_{j+1} \\ \text{and} \quad \mu_j \le y \le z. \end{aligned} \tag{4.2}$$

The solution to this problem is to choose $y = \mu_j$, giving us

$$P_{\max} = \begin{cases} (0.1)\sum_{i=1}^{j}\left(\frac{z-\mu_i}{z}\right) + 0.1\left(\frac{\mu_{j+2}-\mu_{j+1}}{\mu_{j+2}-\mu_j}\right)\left(\frac{z-\mu_j}{z}\right) & ;\ j \le 8 \\ (0.1)\sum_{i=1}^{j}\left(\frac{z-\mu_i}{z}\right) + 0.1\left(\frac{z-\mu_j}{z}\right) & ;\ j \ge 9. \end{cases} \tag{4.3}$$

To find the minimum possible value of P, allocate everybody in decile $(j+1)$ above the poverty line, and as many people as possible from the j^{th} decile to the income level y, $\mu_j \le y \le z$, so as to solve the problem

$$\begin{aligned} P_{\min} = \underset{y}{\text{Min}}\ (0.1) \sum_{i=1}^{j-1}\left(\frac{z-\mu_i}{z}\right) + 0.1\pi\left(\frac{z-y}{z}\right) \\ \text{subject to}\quad \pi y + (1-\pi)\mu_{j-1} = \mu_j \\ \text{and}\quad \mu_j \le y \le z. \end{aligned} \tag{4.4}$$

The solution to this problem is $y = \mu_j$, giving us

$$P_{\min} = \begin{cases} (0.1)\sum_{i=1}^{j}\left(\frac{z-\mu_i}{z}\right) & ;\ j \ge 1 \\ 0 & ;\ j = 0. \end{cases} \tag{4.5}$$

Table 4 presents estimates of these bounds for the year 1975 and the year 2000. Note that for a number of countries (Venezuela, Argentina, Yugoslavia, Mexico, Taiwan, Tunisia, Zambia, Chile, and Korea) the values of $P_{\max}(1975)$ and $P_{\min}(2000)$ are identical. These are countries for which

$$\mu_0 < z < \mu_1$$

in both 1975 and 2000 (which is reflected in the fact that $P_{\min}(1975)$ and $P_{\min}(2000)$ are both zero for these countries). In this case, from (4.4),

$$P_{\max}(1975) = 0.1\left(\frac{\mu_2-\mu_1}{\mu_2}\right)_{1975}$$

where the means are for 1975, and

$$P_{\max}(2000) = 0.1\left(\frac{\mu_2-\mu_1}{\mu_2}\right)_{2000}$$

Table 4. Poverty Gap Ratio Bounds for 1975 and 2000 (per cent)

	Country	$P_{min}(1975)$	$P_{max}(1975)$	$P_{min}(2000)$	$P_{max}(2000)$
1.	Bangladesh	21.5	22.0	9.0	9.8
2.	Ethiopia	...	...	...	...
3.	Burma	...	...	...	...
4.	Indonesia	22.1	22.7	4.2	4.4
5.	Uganda	...	...	...	...
6.	Zaire	...	...	...	...
7.	Sudan	...	...	...	...
8.	Tanzania	...	...	...	...
9.	Pakistan	11.0	11.8	4.1	5.8
10.	India	15.5	15.7	4.8	5.2
11.	Kenya	19.0	19.4	8.1	8.9
12.	Nigeria	...	...	...	...
13.	Philippines	11.2	11.8	1.6	2.5
14.	Sri Lanka	2.4	3.3	0.0	6.1
15.	Senegal	10.3	10.8	5.5	6.1
16.	Egypt	4.8	6.9	0.0	10.7
17.	Thailand	2.9	3.6	0.0	0.4
18.	Ghana	...	...	...	...
19.	Morocco	...	...	...	...
20.	Côte d'Ivoire	2.1	3.3	0.0	2.9
21.	Korea	0.0	4.8	0.0	4.8
22.	Chile	0.0	5.3	0.0	5.3
23.	Zambia	0.0	0.8	0.0	0.8
24.	Colombia	3.0	4.6	0.0	5.0
25.	Turkey	5.2	5.3	0.0	9.0
26.	Tunisia	0.0	3.3	0.0	3.3
27.	Malaysia	2.6	3.8	0.0	11.8
28.	Taiwan	0.0	4.4	0.0	4.4
29.	Guatemala	...	...	...	...
30.	Brazil	1.4	2.3	0.0	5.0
31.	Peru	7.1	10.6	1.6	2.4
32.	Iran	1.8	2.7	0.0	7.0
33.	Mexico	0.0	1.0	0.0	1.0
34.	Yugoslavia	0.0	8.7	0.0	8.7
35.	Argentina	0.0	8.3	0.0	8.3
36.	Venezuela	0.0	7.0	0.0	7.0

Note: ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

where the means are for 2000. Thus the value of P_{max} in each of the two years depends only on the ratio of the mean incomes of the first two deciles in that year. But this is the same as the ratio of the first two decile shares, and this ratio is assumed constant in going from quintile shares to decile shares. Hence the results in Table 4.

The trend in the P index can be significantly different from that in the H index. Thus, comparing $H_{max}(1975)$ with $H_{max}(2000)$, every single country shows a decline in poverty, and the same is true when comparing $H_{min}(1975)$ and $H_{min}(2000)$. However, while the P_{min} comparison between 1975 and 2000 does show a decreasing trend for every country, the P_{max} comparison shows an *increasing* trend for seven countries (Sri Lanka, Egypt, Colombia, Turkey, Malaysia, Brazil, and Iran). For every one of these countries $P_{min}(1975)$ is non-zero while $P_{min}(2000)$ is zero (but there are two countries—Thailand and Côte d'Ivoire—where this is true but the increasing trend is not seen). Clearly the upper bound of the P measure can behave very differently to the upper bound of the H measure—and sole reliance on forecasts of the H measure should be treated with caution.

5 The limited dependent variable problem: poverty projections based on logistic regressions

As we have discussed at length elsewhere (see Anand and Kanbur 1984*c*), it is not a legitimate econometric procedure to regress quintile shares on income, as Ahluwalia (1976) does, without taking account of the fact that cumulative quintile shares are limited dependent variables. There are a number of ways around this problem; we use the method of the logistic transform. Let Q_1, Q_2, Q_3, Q_4, and Q_5 be the five quintile shares, and let

$$
\begin{aligned}
I_{20} &= Q_1 \\
I_{40} &= Q_1 + Q_2 \\
I_{60} &= Q_1 + Q_2 + Q_3 \\
I_{80} &= Q_1 + Q_2 + Q_3 + Q_4
\end{aligned}
$$

be the income shares of the lowest 20, 40, 60, and 80 per cent of the population, respectively. Now I_{20} must lie between 0 and 20, I_{40} must lie between 0 and 40, I_{60} must lie between 0 and 60, and I_{80} must lie between 0 and 80. Applying the logistic transform to these variables,

Table 5. Headcount Ratio Estimates for 1975 and 2000 based on Logistic Regressions (per cent)

	Country	$H_{\log}(1975)$	$H_{\log}(2000)$
1.	Bangladesh	60.8	34.9
2.	Ethiopia	...	...
3.	Burma	...	...
4.	Indonesia	62.5	18.7
5.	Uganda	...	...
6.	Zaire	...	...
7.	Sudan	...	...
8.	Tanzania	...	...
9.	Pakistan	33.7	15.1
10.	India	47.1	19.9
11.	Kenya	48.1	27.6
12.	Nigeria	...	...
13.	Philippines	28.7	6.4
14.	Sri Lanka	10.3	4.4
15.	Senegal	28.7	17.5
16.	Egypt	13.8	3.9
17.	Thailand	22.2	2.4
18.	Ghana	...	...
19.	Morocco	...	...
20.	Côte d'Ivoire	13.8	3.3
21.	Korea	3.8	0.8
22.	Chile	4.6	1.5
23.	Zambia	3.7	2.6
24.	Colombia	13.4	1.8
25.	Turkey	15.1	3.4
26.	Tunisia	4.7	1.1
27.	Malaysia	7.9	1.8
28.	Taiwan	1.9	0.6
29.	Guatemala	...	...
30.	Brazil	8.1	1.4
31.	Peru	14.7	6.1
32.	Iran	8.0	1.6
33.	Mexico	2.2	0.8
34.	Yugoslavia	1.8	0.4
35.	Argentina	1.7	0.6
36.	Venezuela	2.6	0.7

Note: ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

we have

$$I^\star_{20} = \log\left(\frac{I_{20}}{20 - I_{20}}\right)$$

$$I^\star_{40} = \log\left(\frac{I_{40}}{40 - I_{40}}\right)$$

$$I^\star_{60} = \log\left(\frac{I_{60}}{60 - I_{60}}\right)$$

$$I^\star_{80} = \log\left(\frac{I_{80}}{80 - I_{80}}\right).$$

Now we can regress each of the $I^\star$ variables on different functional forms in per capita income in an econometrically consistent way, since the $I^\star$ variables vary from minus ∞ to plus ∞.

We now use the ACC 'base year' observation method to forecast $I^\star_{20}$, $I^\star_{40}$, $I^\star_{60}$, $I^\star_{80}$ in the projection year—i.e. we assume that if the $I^\star$ observation for a country is above (below) the estimated $I^\star$ relationship, then it will remain above (below) this relationship by the same amount throughout. Let hats denote forecast values. Solving for $\widehat{I}$ from $\widehat{I}^\star$, the forecast quintile shares are then given by sequential subtraction:

$$\widehat{Q}_1 = \widehat{I}_{20}$$

$$\widehat{Q}_2 = \widehat{I}_{40} - \widehat{I}_{20}$$

$$\widehat{Q}_3 = \widehat{I}_{60} - \widehat{I}_{40}$$

$$\widehat{Q}_4 = \widehat{I}_{80} - \widehat{I}_{60}$$

$$\widehat{Q}_5 = 100 - \widehat{I}_{80}.$$

These quintile shares are then converted to decile shares in the manner of ACC, and the headcount ratio is interpolated using the ACC formula given in Section 2.

The results are given in Table 5 as $H_{\log}(1975)$ and $H_{\log}(2000)$. These are to be compared with $H(1975)$ and $H(2000)$, in order to see the difference that the logistic transformation makes. In fact, $H(1975)$—q.v. Table 1—and $H_{\log}(1975)$ are very close to each other. For every country the discrepancy is less than or equal to half a percentage point. The discrepancies are somewhat greater for the $H(2000)$—q.v. Table 2—and $H_{\log}(2000)$ comparison: there are four countries with discrepancies larger than one percentage point, but overall these are again small. It is also worth noting that the trend of poverty reduction for each country between 1975 and 2000 is borne out for the $H_{\log}$ comparison as much as for the H comparison.

Overall then, while the logistic transform and subsequent sequential subtraction are clearly econometrically preferable procedures, they do not seem to alter the forecasts of poverty by a great deal. This is because by and large the forecast values of quintile shares remain within the range used for estimation. However, at the extremes there could be a big difference—shown at its most absurd when, in the non-logistic case, the share of the bottom 40 per cent, say, exceeds 40 per cent.

6 Alternative functional forms

In the previous section we argued that the appropriate way of treating I_{20}, I_{40}, I_{60}, and I_{80} as dependent variables in our regressions was to introduce them as logistically transformed variables, i.e. $I^\star_{20}$, $I^\star_{40}$, $I^\star_{60}$, and $I^\star_{80}$. But what about the independent variable, per capita income? Throughout we have introduced this in 'log-quadratic' form, i.e. following Ahluwalia (1976) and ACC we have chosen the functional form of the regressions to be such that the independent variables are log per capita income and the square of log per capita income. As we have argued elsewhere, there is no theoretical reason why such a functional form should be used, and we have experimented with alternative functional forms (see Anand and Kanbur 1984*a*,*b*,*c*).

In what follows we explore the consequences of using

$$I^\star = \alpha + \beta Y + \gamma Y^2$$

and

$$I^\star = \alpha + \beta(1/Y) + \gamma(1/Y)^2$$

as alternative functional forms representing the relationship between cumulative quintile shares and per capita income (Anand and Kanbur 1984*c*). These are estimated from Ahluwalia's (1976) data and, using these estimates, the procedure of the previous section is repeated. The resulting forecasts of headcount ratios for 1975 and 2000 are presented in Table 6.

Comparing the log-quadratic with the quadratic form, i.e. Tables 5 and 6, we see that in 1975 only for four countries is the absolute discrepancy greater than one percentage point. The biggest relative discrepancy is for Thailand, for which the quadratic form forecast is 32 per cent lower than the log-quadratic form forecast. For other countries, the relative discrepancies are by and large below 5 per cent. As might be expected, the discrepancies are much larger for forecasts for the year 2000, where the per capita incomes projected are outside

Table 6. Headcount Ratio Estimates for 1975 and 2000 based on Alternative Functional Forms (per cent)

		Quadratic Functional Form		Inverse Quadratic Functional Form	
	Country	$H_{\log}(1975)$	$H_{\log}(2000)$	$H_{\log}(1975)$	$H_{\log}(2000)$
1.	Bangladesh	61.2	28.1	52.8	52.3
2.	Ethiopia	...	...	...	...
3.	Burma	...	...	...	...
4.	Indonesia	60.2	7.7	75.9	55.6
5.	Uganda	...	...	...	...
6.	Zaire	...	...	...	...
7.	Sudan	...	...	...	...
8.	Tanzania	...	...	...	...
9.	Pakistan	30.7	9.6	39.1	17.2
10.	India	45.8	11.9	53.0	27.6
11.	Kenya	46.4	19.3	47.5	19.8
12.	Nigeria	...	...	...	...
13.	Philippines	27.8	4.7	28.0	4.2
14.	Sri Lanka	10.3	4.0	10.3	3.9
15.	Senegal	29.2	15.9	29.6	14.9
16.	Egypt	13.6	3.6	13.5	3.1
17.	Thailand	15.1	1.9	18.0	1.8
18.	Ghana	...	...	...	...
19.	Morocco	...	...	...	...
20.	Côte d' Ivoire	13.2	3.2	12.6	2.8
21.	Korea	3.7	0.9	3.6	0.7
22.	Chile	4.6	1.7	4.7	1.4
23.	Zambia	3.6	2.5	3.6	2.3
24.	Colombia	12.9	2.0	12.3	1.6
25.	Turkey	14.1	3.6	13.2	2.8
26.	Tunisia	4.5	1.3	4.3	1.0
27.	Malaysia	7.8	2.3	7.4	1.8
28.	Taiwan	1.9	0.7	1.9	0.6
29.	Guatemala	...	...	...	...
30.	Brazil	7.7	1.9	6.4	1.4
31.	Peru	14.7	7.0	14.3	5.8
32.	Iran	8.0	2.2	7.2	1.7
33.	Mexico	2.3	1.0	2.2	0.8
34.	Yugoslavia	1.8	0.6	1.7	0.4
35.	Argentina	1.8	0.9	1.7	0.7
36.	Venezuela	2.7	1.3	2.6	1.0

Note: ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

the range of the regression estimates. Here the absolute discrepancy is greater than one percentage point for seven countries, while the relative discrepancies are much larger. Especially for the fast growing countries with low poverty, relative discrepancies can be well over 25 per cent (Venezuela, Argentina, Yugoslavia, Mexico, Iran, Brazil, etc.). Even for slower growing countries, the relative discrepancy can be large, e.g. Bangladesh (19.4 per cent) and India (40 per cent).

Turning now to the inverse-quadratic functional form, and comparing the values of $H_{\log}(1975)$ in Table 6 with those of $H_{\log}(1975)$ in Table 5, we see that the 1975 estimate for Bangladesh is 8 percentage points lower with the inverse-quadratic form. But the projection for the year 2000 is 18 percentage points *higher* with the inverse-quadratic form (comparing $H_{\log}(2000)$ in Tables 5 and 6). Similar large discrepancies are observed for other countries. Using Table 5 values as a base, the $H_{\log}(1975)$ figures differ by more than 10 per cent for eight of the 26 countries, and this is true for seventeen countries if we compare the $H_{\log}(2000)$ values in Tables 5 and 6. Choice of functional form can, therefore, make a big difference to the projections. This conclusion is all the more serious because, as we have shown elsewhere, the data do not always allow us to select the log-quadratic functional form over the inverse-quadratic form (Anand and Kanbur 1984*c*).

7 Conclusions

The object of this paper has been to examine the robustness of the well-known World Bank-ACC projections of international poverty with respect to various aspects of the methodology underlying them. Of course, every set of projections has to make assumptions in order to simplify a complex reality. However, it is advisable to conduct sensitivity analysis with respect to these assumptions in order to see which ones are crucial to the outcome.

The ACC projections rest on a particular set of assumptions, which we have attempted to identify and clarify in Section 2. At the heart of the methodology is the use of Ahluwalia's (1976) estimates of the Kuznets curve in order to project quintile shares. Poverty is then interpolated from these shares. We have examined the econometric implications of the basic assumption used in the forecasting of quintile shares, namely that a country's quintile share remains a fixed amount above or below the Kuznets curve estimated for that quintile share. We have derived the conditions under which this method will dominate the usual OLS method of projection.

Even if we accept the above forecasting method, we are left with a large number of other assumptions that influence the projections—the method of forecasting per capita income, the interpolation of poverty from the forecast quintile shares, the focus on a particular measure of poverty (headcount ratio), the use of Kravis factors in measuring per capita income, the use of alternative functional forms to estimate the Kuznets curve, etc. For each of these we have produced alternative projections by varying the ACC method in reasonable ways. Thus, for example, the ACC interpolation gives a unique figure of the headcount ratio from the forecast quintile shares, whereas all we can really derive without knowing the distribution within each quintile are lower and upper bounds for the headcount ratio. We show that these bounds can be quite wide, and may even imply a reversal of the trend suggested by the ACC projections. Similarly, exclusive focus on the headcount ratio may be misleading, since if we interpolate the poverty gap measure (which is sensitive not only to the number of poor but also to their average income gap), we find that at least some of the forecast poverty trends are reversed.

When we consider alternative functional forms for the Kuznets curve, and base poverty forecasts on estimates of these alternative forms a big difference is made to the projections in some cases. Thus, when per capita income is introduced in quadratic or inverse-quadratic form in the estimation of the Kuznets curve, in contrast to Ahluwalia's log-quadratic form, we find that although there are no reversals of trend, the actual forecasts for the year 2000 vary greatly. This is particularly significant given our arguments elsewhere that there are no strong grounds in theory or in econometrics to choose between these forms (see Anand and Kanbur 1984*a*,*b*,*c*).

Appendix A

Econometric basis of the ACC projection of quintile shares

The ACC projection methodology assumes that countries run 'parallel' to the Kuznets curve, in the manner described in Section 2. In this Appendix we evaluate their procedure econometrically, and investigate efficient prediction of the dependent variable Q—the income share of a given quintile. We also examine the assumptions under which the ACC procedure may have some justification.

The equation estimated by Ahluwalia (1976) for a cross-sectional data set of n $(= 60)$ countries is

$$Q_i = \alpha + \beta(\log Y_i) + \gamma(\log Y_i)^2 + \delta D_i + \varepsilon_i \qquad i = 1, 2, \ldots, n \quad (A.1)$$

where Q_i is the income share of a given quintile for country i, Y_i is the country's per capita GNP, D_i is a dummy variable for socialist countries, and ε_i is a random error term. Writing the independent variables $(\log Y_i)$, $(\log Y_i)^2$, etc. as the elements of the $(1 \times k)$ row vector $\mathbf{Z}_i'$ (where k is the number of independent variables other than the constant α), and $\boldsymbol{\beta}$ as the $(k \times 1)$ column vector of coefficients, the model (A.1) may be rewritten as

$$Q_i = \alpha + \mathbf{Z}_i'\boldsymbol{\beta} + \varepsilon_i \qquad i = 1, 2, \ldots, n. \quad \text{(A.2)}$$

Using $\boldsymbol{\tau}$ to denote the $(n \times 1)$ vector of 1's, this can be written in matrix notation as

$$\mathbf{Q} = \boldsymbol{\tau}\alpha + \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad \text{(A.3)}$$

where $\mathbf{Q}$ is $(n\times1)$, $\boldsymbol{\tau}$ is $(n\times1)$, $\mathbf{Z}$ is $(n\times k)$, $\boldsymbol{\beta}$ is $(k\times1)$, and $\boldsymbol{\varepsilon}$ is $(n\times1)$. Letting lower-case q_i, z_i denote deviations from their respective means $\overline{Q} = (1/n)\Sigma Q_i$, etc., (A.3) can now be written

$$\mathbf{q} = \mathbf{z}\boldsymbol{\beta} + (\boldsymbol{\varepsilon} - \boldsymbol{\tau}\overline{\varepsilon}). \qquad \text{(A.4)}$$

This is obtained simply by premultiplying (A.3) by the matrix $\mathbf{A} = [\mathbf{I}_n - (1/n)\boldsymbol{\tau}\boldsymbol{\tau}']$ where $\mathbf{I}_n$ is the $(n \times n)$ identity matrix. That is, $\mathbf{q} = \mathbf{AQ}$, $\mathbf{z} = \mathbf{AZ}$, $(\boldsymbol{\varepsilon} - \boldsymbol{\tau}\overline{\varepsilon}) = \mathbf{A}\boldsymbol{\varepsilon}$.

It is easy to show that the ordinary least squares (OLS) estimators of $\boldsymbol{\beta}$ and α in model (A.3) are

$$\widehat{\boldsymbol{\beta}} = (\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'\mathbf{q}$$
$$\widehat{\alpha} = (1/n)[\boldsymbol{\tau}'\mathbf{Q} - \boldsymbol{\tau}'\mathbf{Z}\widehat{\boldsymbol{\beta}}]. \qquad \text{(A.5)}$$

In this framework, we can now state the ACC procedure as follows. Suppose *another* set of observations on the independent variables Z becomes available for country i, say $\mathbf{Z}_i^{\circ\prime}$ [e.g. $\log Y_i(2000)$, $(\log Y_i(2000))^2$, etc.]. Our task is to predict the value of Q_i° associated with this $\mathbf{Z}_i^{\circ\prime}$. Writing the true relationship as

$$Q_i^\circ = \alpha + \mathbf{Z}_i^{\circ\prime}\boldsymbol{\beta} + \varepsilon_i^\circ, \qquad \text{(A.6)}$$

the OLS predictor of Q_i°, $\widehat{Q}_i^\circ$, is simply given by

$$\widehat{Q}_i^\circ = \widehat{\alpha} + \mathbf{Z}_i^{\circ\prime}\widehat{\boldsymbol{\beta}}. \qquad \text{(A.7)}$$

However, the ACC predictor is obtained by adding

$$\Delta = Q_i - \widehat{\alpha} - \mathbf{Z}_i'\widehat{\boldsymbol{\beta}}$$

to the OLS predictor $\widehat{Q}_i^\circ$. Thus the ACC predictor $Q_i^\circ(\text{ACC})$ is given by

$$Q_i^\circ(\text{ACC}) = Q_i + (\mathbf{Z}_i^\circ - \mathbf{Z}_i)'\,\widehat{\boldsymbol{\beta}}. \qquad \text{(A.8)}$$

We can now ask which of the two predictors $\widehat{Q}_i^\circ$ or $Q_i^\circ(\text{ACC})$ is statistically preferable. For this we need to make some assumptions about the error terms ε_j, $j = 1, 2, \ldots, n$, and ε_i°, and we choose to make the following standard ones.

$$\begin{aligned} E(\varepsilon_j) &= 0 \qquad & j &= 1, 2, \ldots, n \\ E(\varepsilon\varepsilon') &= \sigma^2 \mathbf{I}_n \\ E(\varepsilon_j \varepsilon_i^\circ) &= 0 & j &= 1, 2, \ldots, n \qquad \text{(A.9)} \\ E(\varepsilon_i^\circ) &= 0 \\ E[(\varepsilon_i^\circ)^2] &= \sigma^2. \end{aligned}$$

Under these assumptions, if the true model is as described by (A.3), we have the standard result that the OLS predictor (A.7) is best linear unbiased (BLU). In the case of the ACC predictor, the prediction error can be written [equations (A.2), (A.6), (A.8)] as

$$Q_i^\circ(\text{ACC}) - Q_i^\circ = (\mathbf{Z}_i^\circ - \mathbf{Z}_i)'(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}) + \varepsilon_i - \varepsilon_i^\circ. \qquad \text{(A.10)}$$

Therefore,

$$E[Q_i^\circ(\text{ACC}) - Q_i^\circ] = 0$$

since $E(\widehat{\boldsymbol{\beta}}) = \boldsymbol{\beta}$, i.e. the OLS estimator is unbiased under the model (A.3) and (A.9). Thus $Q_i^\circ(\text{ACC})$ is an unbiased predictor, meaning by this that the expectation of the prediction error is zero. But this must imply that $\widehat{Q}_i^\circ$ has a *smaller variance of prediction error* than $Q_i^\circ(\text{ACC})$ and hence a lower mean square error (since $\widehat{Q}_i^\circ$ is the BLU predictor and $Q_i^\circ(\text{ACC})$ is linear in the Q_j's).

In terms of the criterion of efficient prediction, therefore, the simple OLS procedure *dominates* the ACC procedure *if the true model is as described by (A.3) and (A.9).* Thus a justification for the ACC procedure can only be found in a model whose assumptions depart from the standard ones made above.

The model implicit in ACC

We investigate the implications for efficient prediction when (A.2) is replaced by

$$Q_i = \alpha_i + \mathbf{Z}_i' \boldsymbol{\beta} + \varepsilon_i \qquad i = 1, 2, \ldots, n. \quad \text{(A.11)}$$

The assumption here is that each country's relationship differs only in the constant term α_i. Just as (A.2) can be written in matrix notation as (A.3), (A.11) can be stacked as

$$\mathbf{Q} = \boldsymbol{\alpha} + \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad \text{(A.12)}$$

where $\boldsymbol{\alpha}$ is the $(n \times 1)$ vector of α_i's.

This can be transformed into deviations from the mean by premultiplication with the symmetric, idempotent matrix

$$\mathbf{A} = [\mathbf{I}_n - (1/n)\boldsymbol{\tau}\boldsymbol{\tau}']$$

to give the analogue of (A.4), viz.

$$\mathbf{q} = \mathbf{A}\boldsymbol{\alpha} + \mathbf{z}\boldsymbol{\beta} + \mathbf{A}\boldsymbol{\varepsilon}. \tag{A.13}$$

We do not have data on the individual α_i, but can continue to maintain the fiction for estimation purposes that all the α_i's are the same, i.e. $\mathbf{A}\boldsymbol{\alpha} = \boldsymbol{\tau}\alpha$ as in (A.3). In other words, the variable α_i omitted from (A.11) has been *replaced* by the constant α. Estimating $\boldsymbol{\beta}$ by OLS under this assumption, we have as before from (A.5)

$$\widehat{\boldsymbol{\beta}} = (\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'\mathbf{q}$$
$$\widehat{\alpha} = (1/n)[\boldsymbol{\tau}'\mathbf{Q} - \boldsymbol{\tau}'\mathbf{Z}\widehat{\boldsymbol{\beta}}].$$

Substituting the true model for $\mathbf{q}$ from (A.13), and taking expectations

$$\widehat{\boldsymbol{\beta}} = (\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'(\mathbf{A}\boldsymbol{\alpha} + \mathbf{z}\boldsymbol{\beta} + \mathbf{A}\boldsymbol{\varepsilon})$$

and

$$E(\widehat{\boldsymbol{\beta}}) = (\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'(\mathbf{A}\boldsymbol{\alpha}) + \boldsymbol{\beta}. \tag{A.14}$$

Hence,

$$E(\widehat{\boldsymbol{\beta}}) = \boldsymbol{\beta} \text{ if and only if } \mathbf{z}'(\mathbf{A}\boldsymbol{\alpha}) = \mathbf{0}.$$

Thus the standard OLS estimator $\widehat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ for the model (A.3) will provide unbiased estimates of $\boldsymbol{\beta}$ in the model (A.12) if and only if the observations on *each* of the k independent variables Z are uncorrelated with the α_i's.[10] So far as $\widehat{\alpha}$ is concerned, we have

$$\begin{aligned}\widehat{\alpha} &= (1/n)\boldsymbol{\tau}'[\mathbf{Q} - \mathbf{Z}\,\widehat{\boldsymbol{\beta}}]\\ &= (1/n)\boldsymbol{\tau}'[\boldsymbol{\alpha} + \mathbf{Z}(\boldsymbol{\beta} - \widehat{\boldsymbol{\beta}}) + \boldsymbol{\varepsilon}] \qquad \text{from (A.12).}\end{aligned}$$

Hence,

$$E(\widehat{\alpha}) = \overline{\alpha} - (1/n)\boldsymbol{\tau}'\mathbf{Z}(\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'(\mathbf{A}\boldsymbol{\alpha}), \tag{A.15}$$

where $\overline{\alpha} = (1/n)\boldsymbol{\tau}'\boldsymbol{\alpha} = (1/n)\Sigma\alpha_i$.

We are now in a position to evaluate the relative bias of the OLS and ACC projections when the true model is (A.12). Since in this case

$$Q_i^\circ = \alpha_i + \mathbf{Z}_i^{\circ\prime}\boldsymbol{\beta} + \varepsilon_i^\circ,$$

the prediction error of the OLS predictor is

$$\widehat{Q}_i^\circ - Q_i^\circ = \widehat{\alpha} - \alpha_i + \mathbf{Z}_i^{\circ\prime}(\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta}) - \varepsilon_i^\circ$$

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[10]The expression for the variance of $\widehat{\boldsymbol{\beta}}$, $V(\widehat{\boldsymbol{\beta}})$, is unaffected whether or not $\mathbf{z}'(\mathbf{A}\boldsymbol{\alpha}) = \mathbf{0}$, i.e. irrespective of $\widehat{\boldsymbol{\beta}}$ being a biased estimator of $\boldsymbol{\beta}$ in (A.12).

so that

$$E(\widehat{Q}_i^\circ - Q_i^\circ) = (\overline{\alpha} - \alpha_i) + [\mathbf{Z}_i^{\circ\prime} - (1/n)\tau'\mathbf{Z}](\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'(\mathbf{A}\boldsymbol{\alpha}).$$

On the other hand, the bias of the ACC predictor, from (A.10), is given by

$$\begin{aligned} E[Q_i^\circ(\text{ACC}) - Q_i^\circ] &= (\mathbf{Z}_i^\circ - \mathbf{Z}_i)'[E(\widehat{\boldsymbol{\beta}}) - \boldsymbol{\beta}] \\ &= (\mathbf{Z}_i^\circ - \mathbf{Z}_i)'(\mathbf{z}'\mathbf{z})^{-1}\mathbf{z}'(\mathbf{A}\boldsymbol{\alpha}) \qquad \text{from (A.14).} \end{aligned}$$

Now if $\mathbf{z}'(\mathbf{A}\boldsymbol{\alpha}) = \mathbf{0}$, i.e. each of the independent variables Z is uncorrelated with the α_i's, the ACC predictor will be *unbiased* whereas the OLS predictor will have a *bias* of $(\overline{\alpha} - \alpha_i)$. This framework does provide a rigorous justification for the ACC procedure, but the conditions under which it is preferable to OLS projection are seen to be rather special.

We can also compute the variance of the prediction errors of $Q_i^\circ(\text{ACC})$ and $\widehat{Q}_i^\circ$, and determine the conditions under which the latter has a lower mean square prediction error than the former.

Appendix B

Per capita income projections in ACC

ACC Table 1 (pp. 302–303) provides figures for 1975 GNP per capita, measured in 1970 U.S. dollars for the 36 countries under study. ACC Table 2 (pp. 312–313) gives figures for GNP growth rates for these countries for the periods 1960–1975 and 1975–2000. It also shows population growth rates for 1960–1975 and 1975–2000. Given these figures, and using the 1975 per capita GNP figures as base, we can project per capita GNP backwards up to 1960 or forwards up to 2000.[11] However, ACC Table A.2 (p. 335) also gives growth rates of GDP for subperiods within 1960–1975 and 1975–2000:

> Table A.2 shows the growth rates of GDP that were used in our analysis. The projections for 1975–1985 were embodied in a

[11]'The present study was designed to determine the distributional consequences of existing country projections of GNP and population. These have been made by the World Bank in the context of a global analysis of international trade and capital flows. They provide a point of departure (Base Case) from which to consider changes in internal and external policies. The Base Case incorporates changes in GNP growth expected to occur with some improvement in existing policies as well as changes in population growth that can be anticipated from existing demographic trends. Table 2 gives the growth in population and GNP determined on this basis for the period 1975–2000.' (ACC, pp. 311–314).

Table B.1. Alternative Projections of per Capita GNP for 1975 (in 1970 U.S. Dollars)

	Country	$Y(1975)$	$Y^e(1975)$	$Y^{ee}(1975)$	$Y^{eee}(1975)$
1.	Bangladesh	72	62.9	71.9	72.6
2.	Ethiopia	81	80.5	82.3	82.2
3.	Burma	88	87.7	89.2	89.2
4.	Indonesia	90	92.2	81.3	81.1
5.	Uganda	115	114.5	136.0	136.1
6.	Zaire	105	106.5	109.7	109.5
7.	Sudan	112	110.8	114.1	113.9
8.	Tanzania	118	118.9	125.3	124.9
9.	Pakistan	121	122.7	141.4	141.2
10.	India	102	101.1	110.8	110.9
11.	Kenya	168	173.0	179.7	179.0
12.	Nigeria	176	169.0	141.3	140.8
13.	Philippines	182	182.9	180.1	179.6
14.	Sri Lanka	185	186.7	200.8	200.4
15.	Senegal	227	222.8	222.4	222.0
16.	Egypt	238	233.0	245.4	245.3
17.	Thailand	237	237.5	247.7	247.3
18.	Ghana	255	253.3	256.3	256.4
19.	Morocco	266	263.0	252.9	252.1
20.	Côte d'Ivoire	325	320.7	357.3	356.0
21.	Korea	325	327.3	317.8	316.8
22.	Chile	386	385.1	456.2	456.8
23.	Zambia	363	375.1	390.4	390.9
24.	Colombia	352	351.2	341.6	340.6
25.	Turkey	379	375.8	360.8	359.4
26.	Tunisia	425	412.3	350.6	349.5
27.	Malaysia	471	477.5	462.6	462.2
28.	Taiwan	499	487.5	515.7	513.4
29.	Guatemala	497	494.4	481.3	479.5
30.	Brazil	509	516.5	455.3	454.9
31.	Peru	503	510.9	499.9	498.5
32.	Iran	572	584.5	570.3	569.8
33.	Mexico	758	768.3	794.7	794.8
34.	Yugoslavia	828	824.3	812.3	812.2
35.	Argentina	1097	1122.4	1130.5	1129.2
36.	Venezuela	1288	1270.4	1317.1	1312.2

> World Bank Study ['Prospects for Developing Countries, 1978–1985', World Bank (1977)] and have been adapted directly from that work. For 1985–1990, the terminal growth rates of the 1975–1985 period were used, while for the period 1990–2000, the estimates were made directly by the authors of the paper. Four countries, Burma, Uganda, Zaire, and Taiwan, were not a part of the 'Prospects' study and projections for them were adapted from internal World Bank documents (ACC, p. 334).

This passage raises many questions. The growth rates in ACC Table 2 (pp. 312–313) are for GNP while ACC Table A.2 (p. 335) gives growth rates for GDP. Which of these is actually used in the ACC study? Their n. 15 on p. 314, which says of the growth rates in ACC Table 2 that they 'are based on projections to 1985 or 1990 that underlie recent World Bank studies of the world economy (1977, 1979)', suggests strongly that ACC do not distinguish between the two. However, there is inconsistency between the subperiod growth rates for 1975–1980, 1980–1990, and 1990–2000 in ACC Table A.2, and the growth rates for 1975–2000 in ACC Table 2. The growth rate implied for the longer period by ACC Table A.2 is *not* matched by the figures in ACC Table 2. This can be seen from our Table B.2 which provides estimates for $Y(2000)$, per capita income in the year 2000, using each of the subperiod income growth rates from ACC Table A.2, and $Y'(2000)$, per capita income in the year 2000, using the income growth rates for the entire period 1975–2000 given in ACC Table 2 (the population growth rates are all taken from a single source—ACC Table 2). As can be seen from the table, in none of the cases do the two projections match exactly. However, the differences are small—of the order of 2 per cent or less. In our own work we have taken the $Y(2000)$ projections based on the detailed subperiod growth rates.

A further internal inconsistency in the ACC tables is revealed when we see that in ACC Table A.3 they provide us with figures for 1970 per capita GDP in 1970 U.S. dollars. Applying the growth rates listed elsewhere in their paper (Tables 2 and A.2), do we get the per capita GNP figures for 1975 in ACC Table 1? Our Table B.1 lists three alternative estimates for 1975, as well as the actual figures in ACC Table 1, viz. $Y(1975)$. $Y^e(1975)$ is obtained by applying the 1970–1975 growth rate in ACC Table A.2; $Y^{ee}(1975)$ is obtained by applying the 1960–1975 growth rate in ACC Table 2; $Y^{eee}(1975)$ is obtained by applying the 1960–1975 growth rate implied by the three subperiod growth rates in ACC Table A.2. As can be seen from our Table B.1, *none* of these projections matches the actual figures for $Y(1975)$. However, although some of the discrepancies can be large—

Table B.2. Alternative Projections of per Capita GNP for 2000 (in 1970 U.S. Dollars)

	Country	$Y'(2000)$	$Y(2000)$
1.	Bangladesh	119.5	120.4
2.	Ethiopia	116.4	116.2
3.	Burma	97.0	97.1
4.	Indonesia	225.2	220.5
5.	Uganda	126.7	126.7
6.	Zaire	174.2	172.9
7.	Sudan	241.0	239.0
8.	Tanzania	215.0	216.0
9.	Pakistan	220.8	223.3
10.	India	191.5	191.5
11.	Kenya	298.0	301.0
12.	Nigeria	305.9	305.3
13.	Philippines	585.6	589.6
14.	Sri Lanka	308.4	310.9
15.	Senegal	334.5	337.9
16.	Egypt	669.5	770.8
17.	Thailand	662.8	666.8
18.	Ghana	209.8	210.6
19.	Morocco	600.0	600.2
20.	Côte d'Ivoire	651.1	644.6
21.	Korea	1531.7	1534.5
22.	Chile	1141.8	1150.4
23.	Zambia	533.1	529.2
24.	Colombia	1342.6	1335.0
25.	Turkey	1038.3	1041.3
26.	Tunisia	1619.0	1624.0
27.	Malaysia	1525.6	1512.3
28.	Taiwan	1473.0	1472.2
29.	Guatemala	1095.8	1084.4
30.	Brazil	1793.0	1804.2
31.	Peru	1249.7	1261.9
32.	Iran	1797.9	1800.2
33.	Mexico	1875.1	1857.3
34.	Yugoslavia	3056.2	3061.7
35.	Argentina	2570.9	2574.2
36.	Venezuela	3264.5	3295.1

for example, $Y^e(1975)$ for Bangladesh is 12.6 per cent lower than $Y(1975)$—most discrepancies are of the order of 2 to 3 per cent. In our own work, we have simply used the $Y(1975)$ figures.

The discrepancies we have noted may arise for many reasons—the difference between GNP and GDP may be one of these, although it is unlikely that this could account for some of the large discrepancies. However, what is important to note is that we cannot, from the documentation in ACC, arrive at an unambiguous and consistent projection of per capita income for the year 2000—a projection which is crucial for the poverty forecast. The same holds true for projecting backwards (from 1975) to obtain estimates of per capita income in the 'base year'—the year of the survey. Which income growth rates should we use to project backwards—the ones for 1960–1975 in ACC Table 2, or the ones for the three five-year subperiods 1960–1965, 1965–1970, and 1970–1975 in ACC Table A.2? Our Table B.3 shows the difference that these alternative growth rates can make for the 26 countries for which the 'base year' per capita income is required.[12] As is seen, the difference can be substantial. For Bangladesh, Indonesia, Egypt, Côte d'Ivoire, Chile, Tunisia, and Brazil the discrepancy is 10 per cent or more. In our own work we use the projections based on the most detailed subperiod growth rates, i.e. we use $Y(t)$ in Table B.3.

Appendix C

Kravis Factors

ACC Table 1 presents figures on the percentage of population in poverty in 1975 'using Kravis adjustment factors'. In their paper, ACC (p. 304) argue for this as follows:

> Having chosen a poverty line, the next step is to apply it in such a way as to ensure comparability across countries. The use of official exchange rates to define equivalent levels of expenditure

[12] We take levels as referring to mid-year. Thus, for example, for Kenya the survey date is 1969. From 1969 to 1975 is a period of 6 years (75.5 − 69.5, or 74.5 − 68.5). For Bangladesh, the year is given as 1966/67. We take this as falling in the middle of the two years and giving a period up to 1975 of 8.5 years (75.5 − 67, or 74.5 − 66). In calculating $Y'(t)$ we simply apply the 1960–1975 growth rates given in ACC Table 2 to $Y(1975)$. In calculating $Y(t)$, however, we apply the *subperiod* growth rates. Thus for Kenya in 1969 we allow five years of growth at the 1970–1975 rate from ACC Table A.2 and then one year of growth (70.5 − 69.5, or 69.5 − 68.5) at the 1965–1970 rate from ACC Table A.2. For Bangladesh, we allow five years of growth at the 1970–1975 rate and then 3.5 years of growth at the 1965–1970 rate. Throughout, population is taken to grow at the rate given in ACC Table 2. For one country, Zambia, we need to go back to 1959. Here we have simply applied the 1960–1975 growth rate to get $Y'(t)$, or the 1960–1965 growth rate to get $Y(t)$.

Table B.3. Alternative Estimates of GNP in Year of Survey (in 1970 U.S. Dollars)

	Country	Year of Survey t	$Y'(t)$	$Y(t)$
1.	Bangladesh	1966/67	73.8	81.4
2.	Ethiopia	a	...	...
3.	Burma	b	...	...
4.	Indonesia	1971	79.9	72.2
5.	Uganda	b	...	...
6.	Zaire	a	...	...
7.	Sudan	b	...	...
8.	Tanzania	b	...	...
9.	Pakistan	1963/64	93.0	97.9
10.	India	1964/65	89.3	90.9
11.	Kenya	1969	135.2	140.0
12.	Nigeria	b	...	...
13.	Philippines	1971	164.7	162.7
14.	Sri Lanka	1973	178.7	183.9
15.	Senegal	1960	248.0	249.1
16.	Egypt	1964/65	200.3	224.6
17.	Thailand	1962	135.9	136.3
18.	Ghana	a	...	...
19.	Morocco	b	...	...
20.	Côte d'Ivoire	1970	263.8	293.9
21.	Korea	1971[c]	247.5	241.7
22.	Chile	1968	383.4	445.7
23.	Zambia	1959	335.9	328.2
24.	Colombia	1970	312.3	303.7
25.	Turkey	1968[c]	285.9	275.1
26.	Tunisia	1970	357.6	304.1
27.	Malaysia	1970	391.0	378.8
28.	Taiwan	1972	417.5	431.8
29.	Guatemala	b	...	...
30.	Brazil	1970	414.8	365.6
31.	Peru	1970/71	443.8	435.2
32.	Iran	1971[d]	449.6	440.8
33.	Mexico	1969[c]	624.3	644.9
34.	Yugoslavia	1968	598.3	601.9
35.	Argentina	1961	780.3	778.2
36.	Venezuela	1971	1179.6	1214.1

Notes: 1. ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

2. The year of the survey t is as given in ACC Table A.1, p. 333 with the modifications noted in n.8 of our paper.

[a] As in ACC Table A.1, p. 333: 'Not available, distribution taken from Kuznets curve'.

[b] As in ACC Table A.1, p. 333: 'Available data unreliable, distribution taken from Kuznets curve'.

[c] See n.8 of our paper.

[d] As in ACC Table A.1, p. 333: 'Available data unreliable, Venezuela distribution assumed'.

> in different countries does not ensure equivalent levels of real purchasing power. We have attempted to overcome this problem by using 'equivalent purchasing power conversion ratios' estimated by Kravis and associates from data collected by the United Nations International Comparison Project (ICP). Using these ratios, we can convert the per capita GNP levels in each country into GNP per capita measured in dollars of 1970 U.S. prices—hereafter called ICP dollars. The resulting estimates are shown in table 1.

The Kravis factors used are given in ACC Table A.3 (p. 337). They range from a low value of 1.77 for Venezuela to a high value of 3.10 for Indonesia. Although some of the discussion in ACC on this subject is unclear, it *seems* as though their procedure has been to project 1975 decile shares as before, but to calculate mean income of each decile using Kravis adjusted per capita income—which is given for each country in ACC Table A.3.[13] ACC (p. 336) explain their procedure thus:

> At this point, several methods were considered to incorporate the Kravis factors. If the Kravis factors are applied to income per capita before projecting income distributions, then not only do the regressions of the Kuznets curve need to be re-estimated, but in addition, many of the rapidly developing countries, whose incomes are multiplied by Kravis factors, quickly get beyond the range of the regressions and produce implausible results. We considered solving this problem by using Kravis factors that themselves are functions of income per capita, but the difficulties inherent in this approach rendered it impractical. Therefore, we carried out the analysis, the distribution, and the experiments on redistribution, before applying the Kravis factors, making this final transformation on a country basis after all country analysis, but before any global analysis. Thus, we restrict the use of this transformation to providing a means of adding up the world. The Kuznets curve itself was transformed

[13] There seem to be certain inconsistencies within this table, and between this table and ACC Table 1. For example, multiplying 1970 per capita income at official 1970 exchange rates (column 1 of Table A.3) by the Kravis factor (column 2) does not necessarily give what is claimed to be 'Kravis adjusted' 1970 per capita income (column 3). For Bangladesh, $73 \times 2.77 = 202.2 \neq 204$; for Burma, $85 \times 2.69 = 228.7 \neq 230$; for Zaire, $101 \times 2.68 = 270.7 \neq 272$; for Venezuela, $1180 \times 1.77 = 2088.6 \neq 2094$; etc. These are not large differences, but their presence is worrisome. Similarly, we would expect that multiplying the 1975 per capita income figures in ACC Table 1 by the Kravis factors in ACC Table A.3 would give the 'Kravis adjusted' per capita income for 1975 in Table A.3. This is not always so—e.g. for Venezuela, $1288 \times 1.77 = 2279.8 \neq 2286$.

Table C.1. Headcount Ratio Estimates for 1975 and 2000 based on Kravis Factors (per cent)

	Country	$H_K(1975)$	$H_K(2000)$
1.	Bangladesh	63.8	37.8
2.	Ethiopia	...	...
3.	Burma	...	...
4.	Indonesia	58.6	14.7
5.	Uganda	...	...
6.	Zaire	...	...
7.	Sudan	...	...
8.	Tanzania	...	...
9.	Pakistan	43.4	19.1
10.	India	47.0	18.6
11.	Kenya	55.7	37.0
12.	Nigeria	...	...
13.	Philippines	33.1	8.6
14.	Sri Lanka	13.5	4.9
15.	Senegal	34.8	24.3
16.	Egypt	19.6	4.8
17.	Thailand	32.0	2.8
18.	Ghana	...	...
19.	Morocco	...	...
20.	Côte d'Ivoire	25.0	4.6
21.	Korea	4.5	0.9
22.	Chile	10.8	2.1
23.	Zambia	4.9	3.4
24.	Colombia	18.7	2.1
25.	Turkey	19.4	4.2
26.	Tunisia	10.2	1.4
27.	Malaysia	12.2	2.5
28.	Taiwan	2.6	0.8
29.	Guatemala	...	...
30.	Brazil	15.3	1.8
31.	Peru	18.4	6.9
32.	Iran	13.9	2.0
33.	Mexico	3.5	1.2
34.	Yugoslavia	2.5	0.6
35.	Argentina	2.6	0.9
36.	Venezuela	4.3	1.1

Note: ... denotes that an estimate cannot be derived using the ACC method (see Section 2). These countries are included in our tables for ease of comparison with ACC.

at the mean value of the Kravis factor (1.99) for the sample in Ahluwalia.

The paragraph points up a particular problem with the ACC analysis based on the Ahluwalia (1976) regressions—the fact that the latter does not take into account the 'limited dependent variable' nature of the problem. This point was taken up in Section 5. Ignoring this complication for the moment, ACC seem to be proposing two alternatives: (i) to re-estimate relationships between decile shares and Kravis adjusted per capita incomes, and conduct the whole analysis using Kravis adjusted incomes; and (ii) to stick with the official exchange rate incomes until the last stage, and then convert the mean incomes of deciles using Kravis factors. The general thrust of the above paragraph, as well as other discussion in the text, suggests that the second option is the one chosen. But the last sentence in the above paragraph is confusing. It is not clear why such an operation would be necessary if the second option is chosen, and indeed what its significance would be even if the first option were to be chosen.

Assuming that it is the second option that is chosen, let us go on to look at the headcount ratio estimates for 1975 using official exchange rates and Kravis adjustment factors. From ACC Table 1 it can be seen that the headcount ratio for India is the same for both these cases—46 per cent. This is not surprising, since ACC choose their poverty line at the income level corresponding to the 46th percentile in India. For other countries the difference between the two estimates of headcount ratio is given by the extent to which their Kravis factors differ from India's. Thus Indonesia is the only country whose Kravis adjusted headcount ratio is greater than its official exchange rate headcount ratio—this is a simple consequence of the fact that Indonesia is the only country whose Kravis factor is greater than India's.

Table C.1 presents our estimates for the headcount ratio in 1975 and 2000 using Kravis adjustment factors. Of course, for reasons discussed earlier, our estimates will not be exactly the same as ACC's (notice that our estimate of $H_K(1975)$ for India is 47.0 per cent as opposed to 46 per cent). The main discrepancies lie in countries with low poverty. There are eight countries with discrepancies greater than one percentage point—the most dramatic is the case of Mexico, for which the ACC estimate of poverty incidence is 14 per cent (ACC Table 1), while our estimate is 3.5 per cent. We are unable to account for such a large difference. In terms of the *trend* of poverty as measured by the Kravis adjusted headcount ratio, comparison of $H_K(1975)$ and $H_K(2000)$ in Table C.1 shows that poverty will fall, as was the case for the comparison between $H(1975)$ and $H(2000)$ in our Tables 1 and 2.

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